Microwave Driven Convection in a Square Cavity

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The use of microwaves as a volumetrically distributed source of heat has led to a number of applications where rapid heating of a liquid is achieved. Effective coupling of polar solvents with microwaves had led to an increasing interest in using microwaves for synthetic reactions both under low and high pressures (Michael et al., 1991). Other applications include microwaving liquids for purposes of sterilization and heating. In order to maintain product quality, uniform distribution of heat is of paramount importance in these processes.

Natural convection in a fluid with a uniform heat source has been studied by Gartling (1982), Emara and Kulacki (1980), Lambha et al. (1978), John and Reineke (1974), and May (1991). The study of Gartling is illustrative of the typical effects associated with natural convection in the presence of a uniform heat source. The maximum temperature was located in the upper portions of a rectangular cavity and increasing the strength of the heat source term led to secondary flows. May (1991) observed time periodic flows at high Rayleigh numbers.

Microwaves give rise to a spatially distributed source term. Natural convection in a cylindrical container exposed to microwaves was studied by Datta et al. (1991, 1992). The absorbed microwave power was assumed to decay exponentially into the sample following a Lambert law behavior. This assumption is valid for the large sample dimensions used in their study.

We investigate natural convection of a liquid in a square cavity exposed to microwaves at a frequency of 2,450 Mhz. The microwave power, which is influenced by the sample dimension and dielectric properties of the fluid, is deduced by solving Maxwell's equations. With the microwave power as input, the transient temperature and flow fields in the liquid are simulated with the software package FIDAP. Due to the spatial variation of the absorbed microwave power, the temperature distribution is dependent on which face of the cavity is exposed to microwaves. Certain configurations produce smaller variations in temperature and heat up more uniformly than others.

Theory and Solution Technique

Wave propagation

Consider an infinite rod whose cross section forms a square of side L. Suppose this rod is exposed to a plane wave with its electric field oriented along the sample axis, as illustrated in Figure 1. Maxwell's equations govern the propagation of an electromagnetic wave in the medium. Assuming a time dependence of the form $e^{-i\omega t}$, the equation for the spatial distribution of the electric field in an electrically neutral sample is

$$\nabla^2 E + k^2 E = 0, \tag{1}$$

where $k = \alpha + i\beta$. The phase constant α and the attenuation constant β are related to the dielectric properties of the medium and frequency of radiation f by

$$\alpha = \frac{2\pi f}{c}\sqrt{\frac{\kappa'\left(\sqrt{1+\left(\frac{\kappa''}{\kappa'}\right)^2}+1\right)}{2}}$$

and

$$\beta = \frac{2\pi f}{c}\sqrt{\frac{\kappa'\left(\sqrt{1+\left(\frac{\kappa''}{\kappa'}\right)^2}-1\right)}{2}} \tag{2}$$

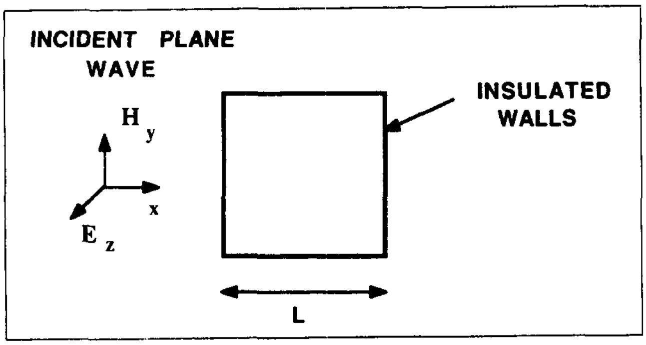

Figure 1. Square cavity exposed to plane waves.

In Eq. 2, c is the velocity of light and κ' and κ'' are the relative dielectric constant and loss respectively. The material dielectric properties are assumed to be independent of temperature. The phase constant α represents the change of phase of the propagating wave and is related to the wavelength of radiation in the medium (λ_m) by $\lambda_m = (2\pi)/\alpha$. The attenuation constant β controls the rate at which the incident field decays into a sample. The quantity β^{-1} is known as the characteristic penetration depth, that is, the distance at which the field intensity decreases to $1/e$ of its incident value. The electric field is calculated by solving Eq. 1 via a Galerkin finite-element method. The details of the analysis are given in an earlier study by Ayappa et al. (1992), where heat conduction in cylindrical and square rods exposed to microwaves was studied. With a knowledge of the electric field, the microwave power absorbed by the medium is,

$$P(r) = \frac{1}{2}\omega\epsilon_0\kappa'' E \cdot E^*, \tag{3}$$

where E^* is the complex conjugate of E and ϵ_0 is the free space dielectric constant.

Fluid flow

The momentum, energy, and continuity equations for a Newtonian Boussinesq fluid are respectively:

$$\rho_0 \frac{\partial v}{\partial t} + \rho_0 v \cdot \nabla v = -\nabla p + \mu \nabla^2 v + g\rho_0[1 - \beta'(T - T_0)], \tag{4}$$

$$\rho_0 C_p \frac{\partial T}{\partial t} + \rho_0 C_p v \cdot \nabla T = \nabla \cdot k \nabla T + P(r), \tag{5}$$

and

$$\nabla \cdot v = 0. \tag{6}$$

In the above equations ρ_0 is the liquid density at the reference temperature T_0, β' is the volume expansion coefficient, v is the velocity, μ the viscosity, p the pressure, g the acceleration due to gravity, k the thermal conductivity, and C_p the specific heat capacity. P is the volumetric microwave power term given in Eq. 3. Sample boundaries are thermally insulated with no slip and no penetration conditions for the flow. We assume the thermal and flow fields to be two-dimensional in nature (that is, invariant along the infinite rod's axis). In addition, in all our calculations, initial temperature in the sample was uniform at 300 K.

The dimensionless number commonly used for convection in the presence of a source term is the modified Grashof number (Gartling, 1982),

$$Gr^* = \frac{\rho^2 g \beta' L^5 \overline{P}}{\mu^2 k}. \tag{7}$$

In this study $\overline{P}$ is the volumetrically averaged microwave power density absorbed by the sample.

In all flow simulations, 576 rectangular biquadratic elements were used. The microwave power was evaluated using a 256 element biquadratic mesh and was interpolated at the gauss points of the 576 element mesh for use with FIDAP. The trapezoidal rule with time step adjustment, with a maximum allowable time step of 1 s, was used for time integration of the temperature and flow equations. Based on a convergence study, a maximum time step of 1 s was found to provide results which were numerically convergent. Material dielectric and thermal properties for oil and water used in this study are listed in Table 1.

Results and Discussion

To investigate the effects of sample size and the direction of microwaves on the uniformity of heating, simulations were carried out for oil in square cavities with sides of 6 and 1 cms. Microwave heating is considered under conditions where uniform plane waves are incident normal to the side, top, and bottom of the cavities. In all cases, according to the usual convention, g is directed from the top towards the bottom of the cavity. Temperature distributions and flow patterns are examined for each configuration as the sample is heated. The influence of changing material properties for a similarly sized sample is studied by heating water in a cavity of side 1 cm. An incident electric field intensity of 4,754.9 V/m was used in all cases considered.

Figures 2, 3, and 4 illustrate the absorbed power contours, temperature contours, and velocity vectors for the $L = 6$ cm oil samples, with microwaves incident as indicated in the figures. The Gr^* number is 8.4×10^7. At 2,450 MHz the penetration depth of microwaves in oil is 37.6 cm and the λ_m is 7.63 cm. Since the sample depth is smaller than the penetration depth, microwaves penetrate into the sample; however, internal reflections give rise to a power peak opposite the face exposed to microwaves.

Temperature contours and velocity vectors are shown in

Table 1. Thermal and Dielectric Properties of Oil and Water

Property	Oil	Water
k (W·m^{-1}·K^{-1})	0.17	0.658
ρ (kg·m^{-3})	920	983.2
C_p (J·kg^{-1}·K^{-1})	2,100	4,181
μ (Pa·s)	5.6×10^{-3}	4.72×10^{-4}
T_0 (K)	323	333
β' (K^{-1})	3.8×10^{-4}	5.344×10^{-4}
κ'	2.567	66
κ''	0.166	3.5

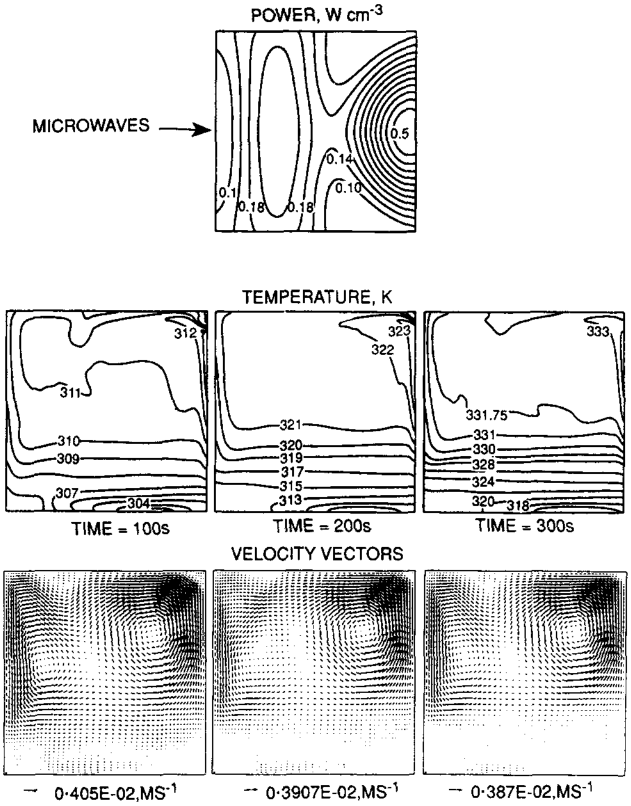

Figure 2. Heating of oil with microwaves incident from the left for $L = 6$ cm.

Maximum power is absorbed near the right face giving rise to two counterclockwise flow cells in the upper half of the sample. Stratification of temperatures in lower half of the sample is seen at 100 s, where heating proceeds primarily by conduction. In upper regions convection dominates, creating a uniformly heated upper zone. Difference between the maximum and minimum temperatures after 300 s is 18.22 degrees.

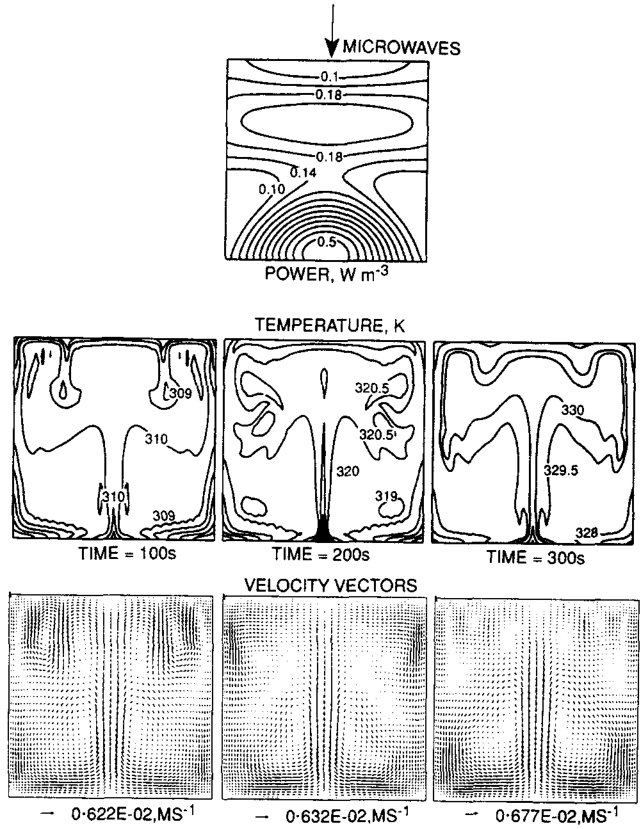

Figure 3. Heating of oil with microwaves incident from the top for $L = 6$ cm.

With power peak at the bottom of the cavity hot fluid is continuously being convected to cooler regions at the top. Temperature gradients within the sample are smallest of three configurations and temperature span is less than a few degrees throughout the heating period. Convection plays a dominant role in lowering temperature gradients within the sample.

Figure 2, for microwaves incident from the left side. The presence of the hot spot at the right, gives rise to two large convection cells in the upper portions of the cavity. This causes fluid to flow from the hotter regions of higher power absorption to the cooler, low power deposition regions at the left face. By 100 s a strong stratification in temperature is observed. There exists a lower layer where heat transfer occurs primarily by conduction, and a convection-dominated upper region. This stratification is still present at 300 s, a time at which the difference between the maximum and minimum temperatures is about 18 degrees.

Figure 3 illustrates heating for microwaves incident from the top. Here the power peak located at the bottom center of the sample drives flow in the cavity. Convection plays a strong role in lowering temperature gradients in the sample. At 300 s there is about a 6 degree difference between the maximum and minimum temperatures. The fluid heats up uniformly, the coldest regions being at the corners of the cavity.

In Figure 4 the microwaves are incident from below and the power peak is now located at the top of the sample. The higher heating rate at the top of the sample quickly stratify the temperature with a well mixed zone in the middle. At 300 s there is a difference of about 40 degrees between the maximum and minimum temperatures.

Note, that in spite of the three configurations having the same Gr^* number, location of the power peak can have a markedly different effect on convection. A useful measure of uniform heating is the difference between maximum and minimum temperatures in the sample. Figure 5 illustrates this difference in temperatures, as heating proceeds, for the different configurations considered. For a fixed sample size, since the amount of power deposited is the same and the sample boundaries are insulated, the volumetrically averaged temperature rise is independent of the direction of microwaves. The spatial temperature variation during heating reflects the role of convection for different configurations. The greatest variation in temperature is for microwaves incident from the bottom, where the power maximum is located at the top of the sample. Here, convection plays the smallest role and regions of higher power absorption continuously heat up faster than regions of low power absorption. Counterintuitive to common thought, the most uniform heating is observed for microwaves incident from the top. Here the power peak is located at the bottom, and convection plays a dominant role. This effect will not be detected with a Lambert law formulation, where the absorbed power is assumed to decay exponentially into the sample. Figure 5 illustrates that the direction of microwaves has a strong influence on the uniformity of temperature in the 6 cm sample.

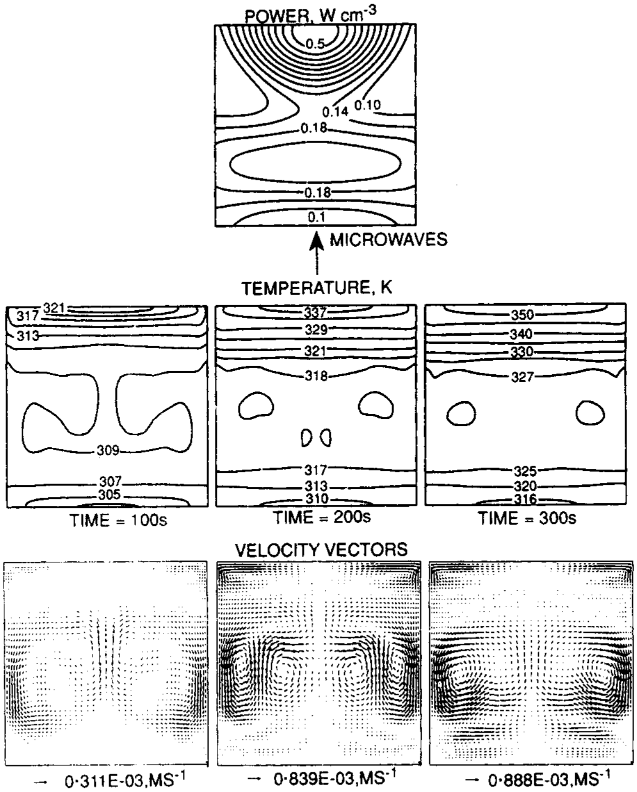

Figure 4. Heating of oil with microwaves incident from the bottom for $L = 6$ cm.

Contrast this situation with Figure 3 where microwaves are incident from the top. Here, maximum power is deposited at the top of the sample, giving rise to the largest temperature gradients among the three configurations. Convection plays a small role in diminishing temperature gradients. Difference between the maximum and minimum temperatures continuously gets larger as sample is heated (see Figure 5) and is 40 degrees at 300 s.

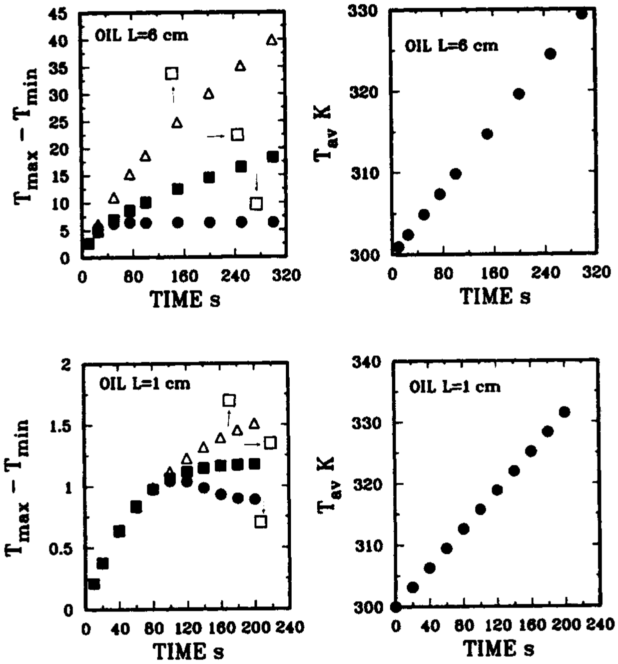

Figure 5. Effect of sample configuration on $T_{max} - T_{min}$, in the sample, as heating proceeds is illustrated here.

Arrows indicate the face of the cavity being exposed to microwaves. For the 6 cm sample, uniformity of heating is strongly influenced by which face of the cavity is exposed to microwaves. When microwaves are incident from above, the power peak is at bottom of sample and convection plays a significant role in reducing temperature gradients. In 1 cm sample the power absorbed has little spatial variation and temperature gradients for the three configurations differ by less than a degree. Average temperature rise which is independent of microwave direction is plotted as a function of heating time.

When using microwaves for sterilization of liquids, uniformity of temperature is crucial to the success of the process. With a knowledge of the power distribution, judicious positioning of the sample can greatly reduce temperature gradients during heating.

Heating for the 1 cm sample of oil for all three configurations is shown in Figure 6. Temperature contours and velocity vectors are shown at 200 s. The power distribution, despite the presence of a weak maximum, is very uniform within the sample resulting in a uniform temperature distribution. The Gr^* number here is 1.8×10^4. For microwaves incident from the left, hot fluid is at the top right corner, and for bottom incidence, the temperatures are stratified. Flow cells formed in the initial stages of heating, persist through the heating period. Figure 5 illustrates the difference between the maximum and minimum temperatures as heating proceeds. Since the spatial variation of absorbed power is less, the direction of microwaves has a smaller effect on heating. The temperature span within the sample for all three configurations is less than a few degrees.

Figure 7 illustrates the heating of a 1 cm sample of water. In water $\beta^{-1} = 9.05$ cm, $\lambda_m = 1.51$ cm and $Gr^* = 4.1 \times 10^7$. At $L = 1$ cm, the intensity of absorbed power and the heating rates for water are significantly greater than those of oil. Two strong power peaks are observed and, for incidence from the left, this results in two large flow cells in the upper region with a conduction-dominated lower region. For incidence from top, the maximum power at the bottom creates two large flow cells that extend nearly to the top of the sample; heating is most uniform in this configuration. Notice however, that the peak in the upper regions of the sample does inhibit good mixing in this case. For incidence from the bottom, the stronger power peak is now located at the top, resulting in a thicker thermally stratified layer. The weaker peak at the bottom gives rise to smaller flow cells. Temperature gradients are the largest for this configuration and after 14.6 s of heating the difference between maximum and minimum temperatures is 41.34 degrees.

Conclusions

Microwaves give rise to a volumetrically distributed heat source. The location, intensity, and number of power peaks influences the uniformity of temperature in the liquid. In the square cavities studied, greater the spatial variation of microwave power, stronger is the influence of the direction of microwaves on uniformity of heating. The configuration that gives the most uniform heating is one where the power maximum is located toward the bottom of the cavity. Here convection plays the largest role in lowering the temperature span in the sample. When power peaks are located at the sides or toward the top of the sample, convection plays a smaller role,

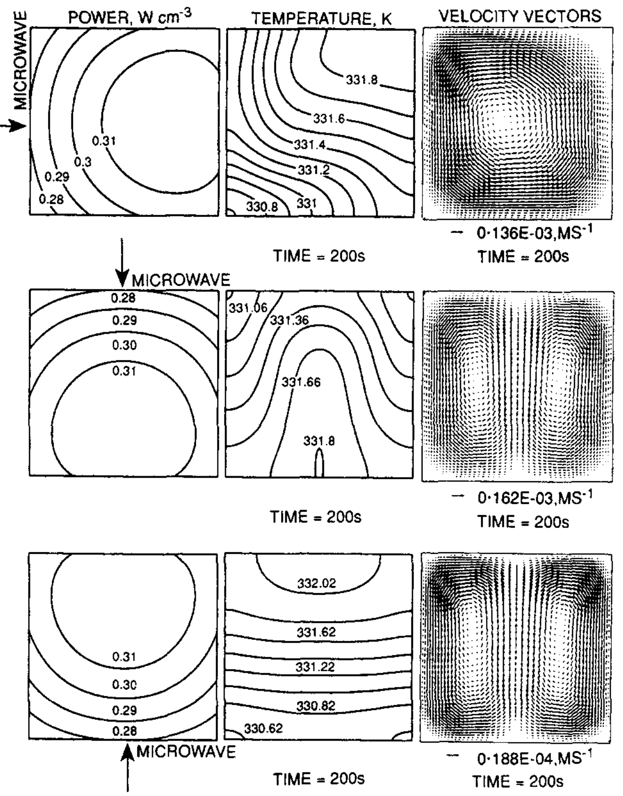

Figure 6. Heating of oil with microwaves incident from the left, top, and bottom for $L = 1$ cm.

Spatial variation of power is small and direction of incidence has little influence on temperature gradients within sample.

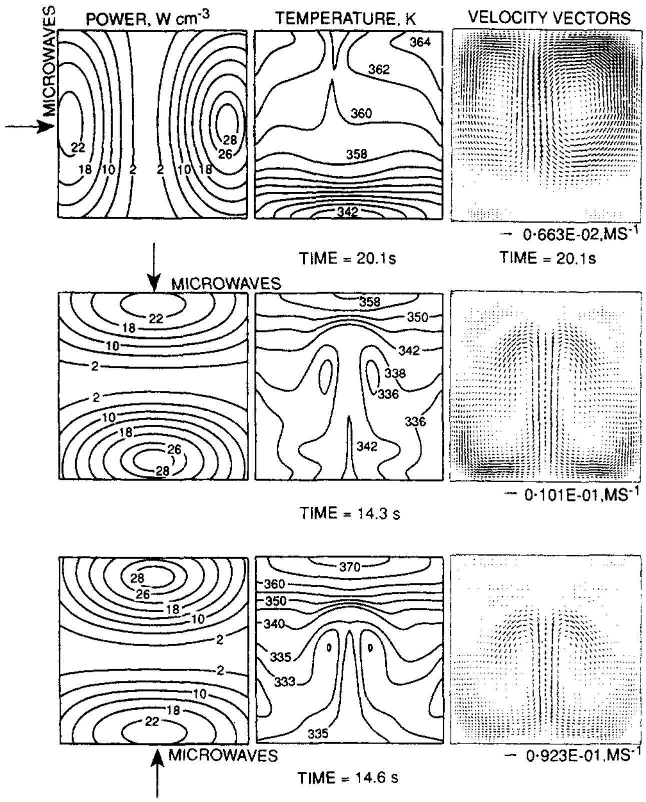

Figure 7. Heating of water with microwaves incident from the left, top, and bottom for $L = 1$ cm.

Positioning of two power peaks influences heating in sample. Temperature gradients are minimum when strongest power peak is at bottom of sample.

and the sample heats up less uniformly. For cases where the power distribution is more uniform, the temperatures are not strongly influenced by the direction of microwaves.

Acknowledgment

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